D. Binomial tables

- 1. Extensive tables have been developed to solve binomial experiments. See Table 1 page ST 1.
- 2. Below is a table for a two trial (n = 2) experiment and some relevant probabilities.
- 3. Note the distribution for the page 46 coin problem is under the .5 column.
- 4. If the probability of a defective part is .05, then getting 2 out of 2 defects would be .0025 or .25%.

Probability of x successful outcomes given the following probability values (p) and trials (n)										Note: A binomial		
Х	0.0500	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.9500	table has 2 defining characteristics, n
0	0.9025	0.81	0.64	0.49	0.36	0.25	0.16	0.09	0.04	0.01	0.0025	and p.
1	0.0950	0.18	0.32	0.42	0.48	0.50	0.48	0.42	0.32	0.18	0.1950	Note: For this
2	0.0025	0.01	0.04	0.09	0.16	0.25	0.36	0.49	0.64	0.81	0.9025	table, n = 2.

E. The shape of binomial distributions

- 1. Distributions are symmetrical when P(x) = .5. High or low probabilities have highly skewed distributions.
- 2. When the $p(x) \neq .5$, the distribution is skewed and a larger n will result in a more symmetrical distribution.

III. The Poisson distribution

- A. A Poisson distribution is similar to a binomial distribution except the P(x) must be small. A Poisson distribution is defined by only 1 characteristic, its mean. The distribution is highly skewed to the right.
- B. Events related to time, such as customers arriving per 5-minute periods, often follow a Poisson distribution.
- C. The mean is needed when using a Poisson distribution.

$$\mu = E(x) = \sum [x \cdot P(x)]$$
 (see page 52)

- D. A Poisson distribution may be determined with a formula or looked up in a table.
 - 1. Calls per 15 minute period to Linda's repair facility follow a Poisson distribution with $\mu = 1.0$. What is the probability of exactly three service calls being received in a randomly selected 15-minute period?

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

$$P(3) = \frac{(1^3)2.7183^{-1}}{3!} = \frac{(1)(0.3679)}{6} = 0.0613$$

See table below

IV. The Poisson approximation of the binomial probability distribution

- A. A Poisson distribution is often used to approximate a binomial distribution for problems such as errors on a typed page, circuit board defects, and customers bouncing checks at Linda's Video Showcase.
- B. This is done to save the time and money necessary to solve extensive binomial experiments.
- C. These two distributions have similar skewness provided the number of trials is large $(n \ge 30)$ and the probability of occurrence (p) is small (either np or nq < 5).
- D. The mean for a Poisson approximation of a binomial is $\mu = np$ (n = trials and p is the probability of an event).
- E. Recent observations revealed that 4 of 40 items purchased by customers are returned. This is a binomial problem with a sample mean of P(s) = 4/40 = .10. Determining the entire distribution by using the binomial formula 40 times would be a tremendous task.
 - 1. Using the Poisson approximation with a sample mean of .10 for μ is much easier. Its use is appropriate as $n \ge 30$ and np < 5 (40 × .1 = 4). The above formula yields a probability of 0 returns equal to .904837.
 - 2. Using a Poisson distribution table to solve this problem only requires locating the appropriate outcome (value of x) under the appropriate mean. μ = .1 and x = 0 \rightarrow .9048 (see Table 2 page ST 2)

	Probability of x outcomes given the following population means											
X	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00		
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679		
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659	0.3679		
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647	0.1839		
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494	0.0613		
4	2	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111	0.0153		
5				0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031		
6							0.0001	0.0002	0.0003	0.0005		
7										0.0001		